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PARAMETRIC STUDIES ON DIRECT CONTACT EVAPORATION OF A DROP IN AN IMMISCIBLE LIQUID

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Abstract—This paper deals with a theoretical analysis of direct contact evaporation of a drop moving in a stagnant column of an immiscible liquid. The non-dimensional parameters governing the motion and heat transfer are identified and a study of the effects of the variation of these parameters is made. The agreement between the predicted results and the available experimental values is reasonably good.

NOMENCLATURE

- B dimensionless radius, R/R_0
- C_D coefficient of drag
- C_p specific heat of continuous liquid phase [J kg^{-1} K^{-1}]
- Fr instantaneous Froude number, $u^2/2Rg$
- Fr_0 initial Froude number, $u_0^2/2R_0g$
- g gravitational acceleration [m s^{-2}]
- G density ratio, ρ_b/ρ
- h instantaneous heat transfer coefficient [W m^{-2} K^{-1}]
- h_{fg} latent heat of evaporation of dispersed liquid phase [J kg^{-1}]
- Ja system Jakob number, $\rho C_p(T_c - T_d)/\rho_v h_{fg}$
- k thermal conductivity of continuous liquid phase [W m^{-1} K^{-1}]
- m mass of liquid fraction in the two-phase bubble [kg]
- m_0 total mass of two-phase bubble [kg]
- Nu instantaneous Nusselt number, $2hR/k$
- Pe instantaneous Péclet number, $2Ru/\alpha$
- Pe_0 initial Péclet number, $2R_0u_0/\alpha$
- Pr Prandtl number, $\mu C_p/k$
- R instantaneous radius of evaporating drop [m]
- R_0 initial radius of drop [m]
- Re instantaneous Reynolds number, $2\rho uR/\mu$
- Re_0 initial Reynolds number, $2\rho u_0R_0/\mu$
- RRC density ratio, ρ_1/ρ
- RRD density ratio, ρ_v/ρ_1
- St system Stefan number, $C_p\Delta T/h_{fg}$
- t time [s]
- T_c temperature of continuous liquid phase [K]
- T_d boiling point of dispersed liquid phase [K]
- ΔT temperature difference, $(T_c - T_d)$
- u instantaneous velocity of evaporating drop [m s^{-1}]
- u_0 initial velocity of the drop [m s^{-1}]
- U dimensionless velocity, u/u_0
- x distance along the column [m]
- X dimensionless distance along the column, x/R_0.

Greek symbols

- α thermal diffusivity of continuous liquid phase [m^2 s^{-1}]
- μ viscosity of continuous liquid phase [kg m^{-1} s^{-1}]
- ρ density of continuous liquid phase [kg m^{-3}]
- ρ_b average density of evaporating drop [kg m^{-3}]
- ρ_l density of dispersed liquid phase [kg m^{-3}]
- ρ_v density of dispersed vapour phase [kg m^{-3}]
- τ dimensionless time, gt/u_0.

INTRODUCTION

DIRECT contact heat transfer between two immiscible liquids has the advantage of eliminating metallic heat transfer surfaces which are prone to corrosion and fouling. The main features of direct contact heat exchangers are: relative simplicity of design, less scaling problems, higher rates of heat transfer, convenient separation of the fluids and ability to operate at relatively small temperature driving forces. The practical applications are found in water desalination, geothermal heat recovery, ocean thermal energy conversion and thermal energy storage systems. A critical review of the work carried out in direct contact heat transfer is given by Sideman [1].

Considerable work has been done to study the process of single drop evaporation in a stagnant column of immiscible liquid. Sideman and Taitel [2] developed an analytical expression for the Nusselt number by solving the energy equation assuming potential flow around a sphere filled with vapour at the top and the unevaporated liquid at the bottom. They ignored heat transfer to the vapour and assumed that the thermal resistance was negligible in the liquid inside the two-phase droplet. Simpson *et al.* [3] observed that oscillations of the droplet induced sloshing of the liquid inside the droplet, so that a thin film effectively coated the entire interior surface of the droplet. Pinder [4] predicted the interfacial areas and the vapour open angle when a liquid drop evaporates in an immiscible liquid. Motion of an evaporating drop in an immiscible liquid has been studied theoretically and experimentally by Selecki and Gradon [5]. Mokhtarzadeh and El-Shirbini [6] presented a theoretical analysis of evaporation of single drops of pentane and butane in a stagnant column of distilled water. They observed that use of the existing relations for the determination of the Nusselt number did not give satisfactory agreement with the experimental results under all operating conditions. Prakash and Pinder [7], Adams and Pinder [8], and Smith *et al.* [9] have developed experimental correlations for the Nusselt number for different pairs of immiscible liquids.

In spite of a large number of experimental investigations, a generalized theoretical analysis of direct contact evaporation in immiscible liquids is found to be lacking in the literature. A study of the various parameters governing the evaporation process is essential to understand the phenomenon of direct contact evaporation. The present investigation deals with a parametric study of single drop evaporation in an immiscible liquid. It also aims at the prediction of the evaporation process of a drop in any immiscible liquid.

THEORETICAL ANALYSIS

The governing equations for the motion and heat transfer during the evaporation of a spherical drop in a column of stagnant immiscible liquid, maintained at a uniform temperature, are shown below.

A momentum balance of the moving system gives

$$\frac{d}{dt}\left(m_0 u + \frac{2\pi R^3 \rho u}{3}\right) = \tfrac{4}{3}\pi R^3 g(\rho - \rho_b) - \frac{\pi R^2 \rho u^2 C_D}{2}. \tag{1}$$

Hence

$$\frac{du}{dt} = \frac{1}{(4\pi R^3/3)[\rho_b + (\rho/2)]}\left[\tfrac{4}{3}\pi R^3 g(\rho-\rho_b) - \frac{\pi R^2 \rho u^2 C_D}{2} - 2\pi R^2 \rho u \frac{dR}{dt}\right]. \tag{2}$$

The bubble position and velocity are governed by the relation

$$\frac{dx}{dt} = u. \tag{3}$$

A combination of energy and mass balances results in the following relation for the bubble growth rate [6]

$$\frac{dR}{dt} = \frac{(\rho_l - \rho_v)}{\rho_l \rho_v h_{fg}} h(T_c - T_d). \tag{4}$$

The initial conditions are:

$$\text{at} \quad t = 0, \quad u = u_0, \quad x = 0 \quad \text{and} \quad R = R_0. \tag{5}$$

The evaporation is complete when

$$R = R_0(\rho_l/\rho_v)^{1/3}. \tag{6}$$

Equations (2)–(4) are non-dimensionalized using the following transformations

$$\begin{aligned} &X = x/R_0, \quad U = u/u_0, \quad B = R/R_0, \quad \tau = gt/u_0,\\ &Re = 2\rho u R/\mu, \quad Re_0 = 2\rho u_0 R_0/\mu,\\ &Pe = 2uR/\alpha, \quad Pe_0 = 2u_0R_0/\alpha,\\ &Fr = u^2/2Rg, \quad Fr_0 = u_0^2/2R_0 g,\\ &Nu = 2hR/k, \quad St = C_p \Delta T/h_{fg},\\ &RRC = \rho_l/\rho, \quad RRD = \rho_v/\rho_l,\\ &Ja = \rho C_p \Delta T/\rho_v h_{fg} = St/(RRD \cdot RRC),\\ &G = \rho_b/\rho = RRC/B^3. \end{aligned} \tag{7}$$

The resulting dimensionless equations are:

$$\frac{dU}{d\tau} = \frac{(1-G)-(1/B)[0.75 C_D Fr_0 U^2 + 1.5U(dB/d\tau)]}{(G+\frac{1}{2})}, \tag{8}$$

$$\frac{dX}{d\tau} = 2U\, Fr_0, \tag{9}$$

$$\frac{dB}{d\tau} = 2(1-RRD)\frac{Nu\, Ja\, Fr_0}{B\, Pe_0}. \tag{10}$$

The initial conditions are:

$$\text{at} \quad \tau = 0, \quad X = 0, \quad U = 1 \quad \text{and} \quad B = 1. \tag{11}$$

The final condition at the completion of evaporation is

$$B = RRD^{-1/3}. \tag{12}$$

Equations (8)–(10) are non-linear and hence closed-form solutions cannot be obtained. The above three differential equations are simultaneously integrated by the variable step Runge–Kutta method using the continuous system modelling program (CSMP) in a 370/155 IBM computer. Relations for coefficient of drag and Nusselt number are given below.

As no exact relation for C_D for the case of an evaporating and moving drop in an immiscible liquid is available, the relation for the coefficient of drag, applicable to motion of air bubbles in water, given by Haberman and Morton [10] is used in this work. The coefficient of drag is expressed as a function of Reynolds number.

Some relations for the instantaneous Nusselt number are available in the literature. Sideman and Taitel [2] developed the following relation for the instantaneous Nusselt number based on their theoretical analysis

$$Nu = 0.272 Pe^{0.5}. \tag{13}$$

Sideman and Taitel [2] also obtained the following experimental correlation for the instantaneous heat transfer coefficient

$$h = \frac{a_1}{\Delta T}\frac{(1-m/m_0)^{a_2}}{[1 + a_3(1-m/m_0)^{a_4}]}, \tag{14}$$

where a_1, a_2, a_3 and a_4 are given for different experimental conditions. Simpson *et al.* [3] obtained the following relation for the instantaneous heat transfer coefficient for a butane drop evaporating in water or brine

$$h = \frac{2570B^{1/6}}{1+0.206B^{5/12}}. \quad (15)$$

Equation (15) has been obtained using experimental data for single butane drops of about 3.75 mm diameter and a temperature driving force ranging from 1.5 to 8 K. Smith *et al.* [9] suggest the following relation for the instantaneous Nusselt number:

$$Nu = \gamma\, Re^{y}\, Pr^{1/3}. \quad (16)$$

The value of γ depends on the initial diameter of the drop and the value of exponent y.

Equation (13) is the only general relation available in the literature for the determination of the instantaneous Nusselt number. But the agreement of this relation with the experimental results is not consistent at different temperature differences and this has been observed by Mokhtarzadeh and El-Shirbini [6] in their theoretical work. The experimental results available in the literature indicate the dependence of Nusselt number on temperature difference. Sideman and Taitel [2] have obtained experimentally the instantaneous heat transfer coefficients for three different systems with various drop diameters and at several temperature differences and percentages of evaporation. A regression analysis is carried out using their experimental values and the following correlation is obtained

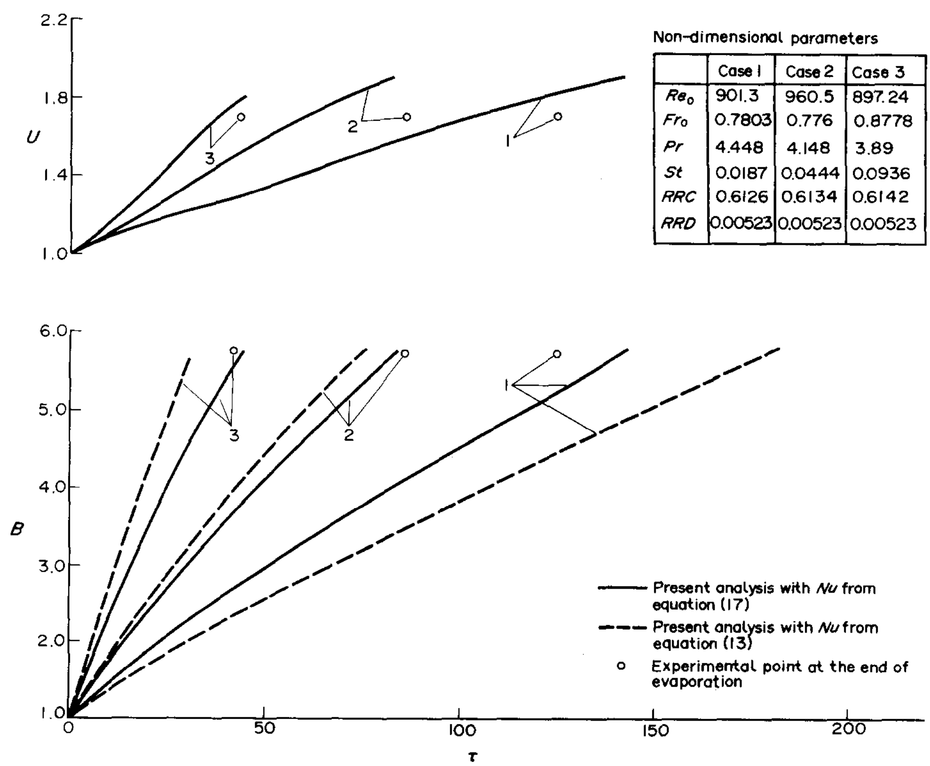
$$Nu = 0.64Pe^{0.5}\, Ja^{-0.35}. \quad (17)$$

This relation is used in this work for the calculation of the instantaneous Nusselt number.

The temperature of the continuous phase liquid is assumed to be constant throughout the column. The effect of the hydrostatic head has been neglected and the boiling point of the dispersed phase liquid is assumed to remain constant during evaporation. The effect of the hydrostatic head is not considerable for smaller drops or for high temperature differences. The properties of the fluids at the appropriate temperatures are obtained from the published literature.

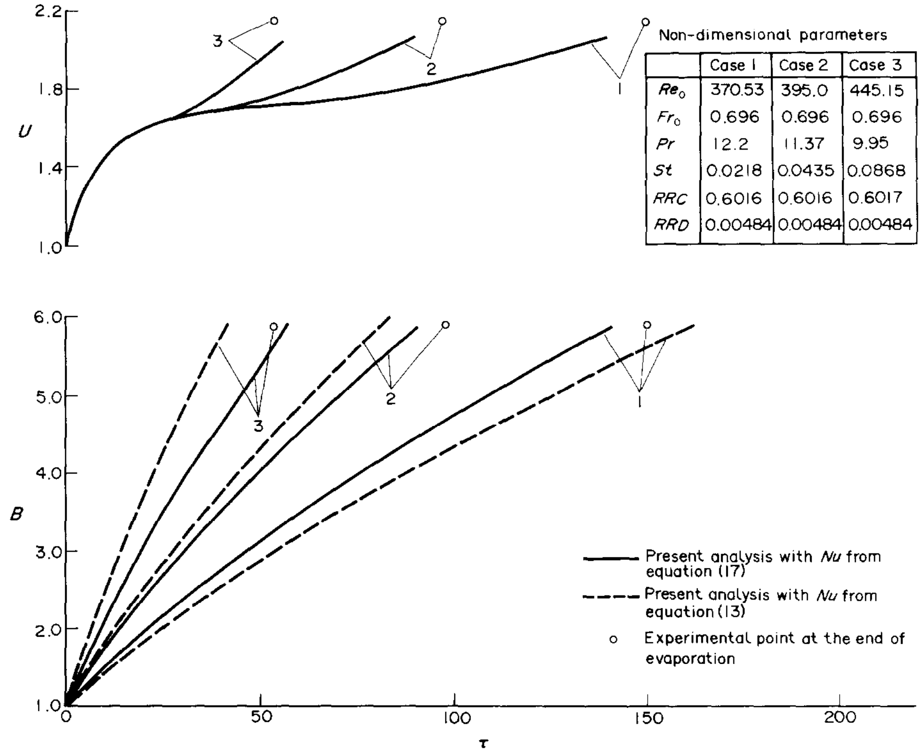
RESULTS AND DISCUSSION

The theoretical model is verified by comparing the results with some of the available experimental results. Figure 1 shows a comparison of the theoretical results with the experimental results of Sideman and Taitel [2], when single pentane drops of 3.6, 3.62 and 3.2 mm diameter evaporate along a column of distilled water at $\Delta T = 1.6$, 3.8 and 8 K, respectively (Run Nos. 1, 7 and 14). Figure 1 shows the variation of radius ratio with the dimensionless time when relations (13) and (17) are used for the evaluation of Nusselt number. The figure also shows the experimental time for complete evaporation. The maximum deviation of the predicted time from the experimental time is as high as 45% when relation (13) is



	Case 1	Case 2	Case 3
Re_0	901.3	960.5	897.24
Fr_0	0.7803	0.776	0.8778
Pr	4.448	4.148	3.89
St	0.0187	0.0444	0.0936
RRC	0.6126	0.6134	0.6142
RRD	0.00523	0.00523	0.00523

FIG. 1. Comparison of the theoretical model with the experimental results [2].



	Case 1	Case 2	Case 3
Re_0	370.53	395.0	445.15
Fr_0	0.696	0.696	0.696
Pr	12.2	11.37	9.95
St	0.0218	0.0435	0.0868
RRC	0.6016	0.6016	0.6017
RRD	0.00484	0.00484	0.00484

FIG. 2. Comparison of the theoretical model with the experimental results [3].

used for the determination of the Nusselt number. The predicted time using relation (17) for Nu agrees fairly well with the experimental time and the maximum deviation is about 15%. The variation of the velocity of the evaporating drop with time is also shown in Fig. 1. The predicted final velocity is higher than the experimental final velocity in all three cases and the maximum deviation is about 12%.

Figure 2 compares the predicted results with the experimental results of Simpson *et al.* [3] for a butane drop of about 3.75 mm in diameter evaporating along a column of distilled water at $\Delta T = 2$, 4 and 8 K, respectively. Figure 2 shows that the time predicted by the model using relation (13) does not agree satisfactorily with the experimental time and the maximum deviation is about 25%. The predicted time

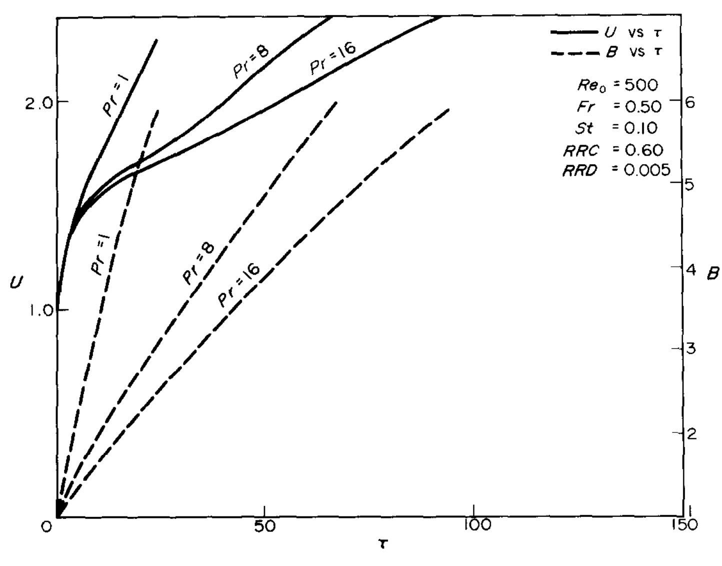


FIG. 3. Effect of Pr on U and B.

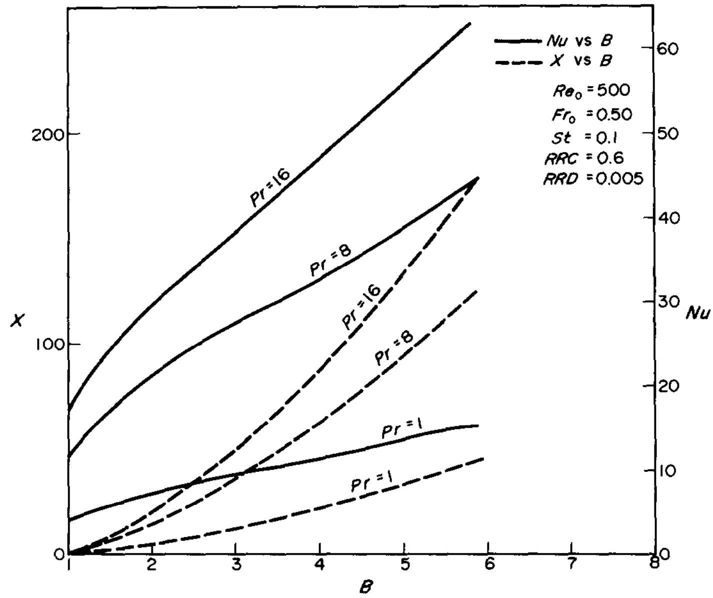


FIG. 4. Effect of *Pr* on *Nu* and *X*.

using relation (17) is close to the experimental time and the maximum deviation is less than 10%. Figure 2 also shows the variation of velocity with time and the predicted final velocity is lower than the experimental final velocity and the maximum deviation is about 5%.

The comparison of the theoretical results with the experimental data shows that the maximum deviation is limited to about 15%. This difference may be attributed to the simplifying assumptions in the model and to the use of an approximate relation for C_D.

In order to study the effects of the various parameters in direct contact evaporation of a drop in an immiscible liquid, a parametric analysis is carried out. The ranges of the parameters used in the analysis are:

Pr	1.0–16.0,
Re_0	250–1000,
Fr_0	0.25–1.0,
St	0.02–0.2,
RRC	0.3–1.5,
RRD	0.0025–0.01,

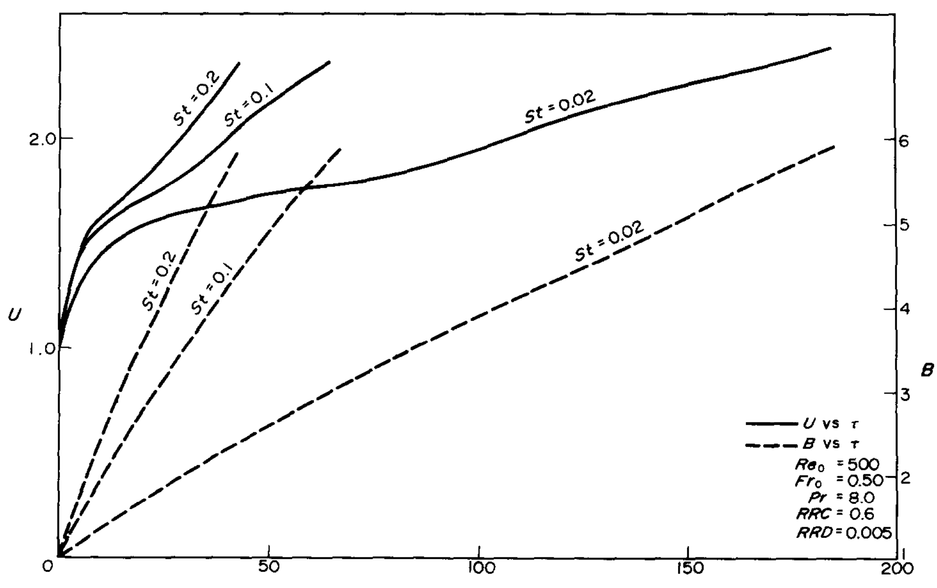


FIG. 5. Effect of *St* on *U* and *B*.

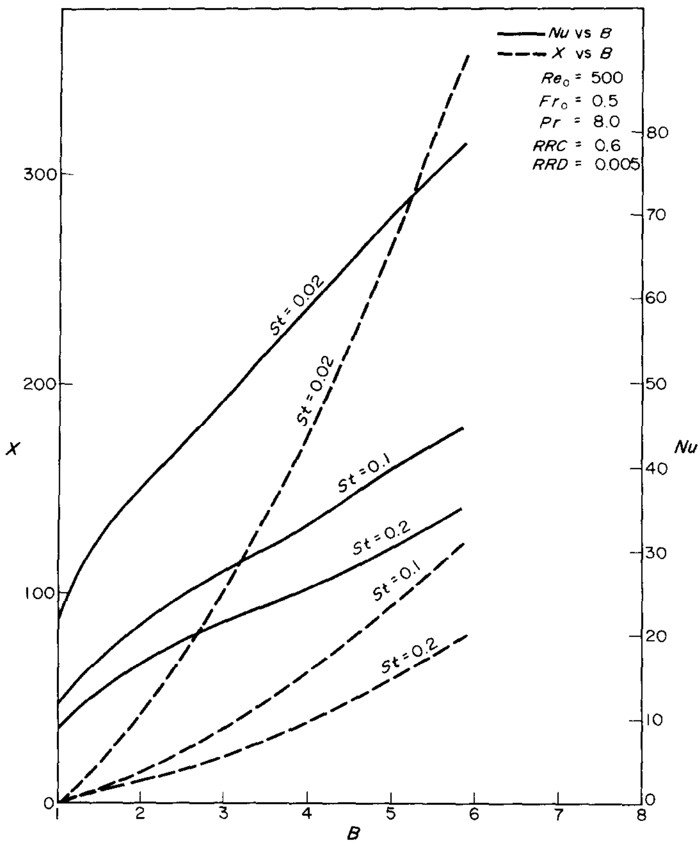


FIG. 6. Effect of St on Nu and X.

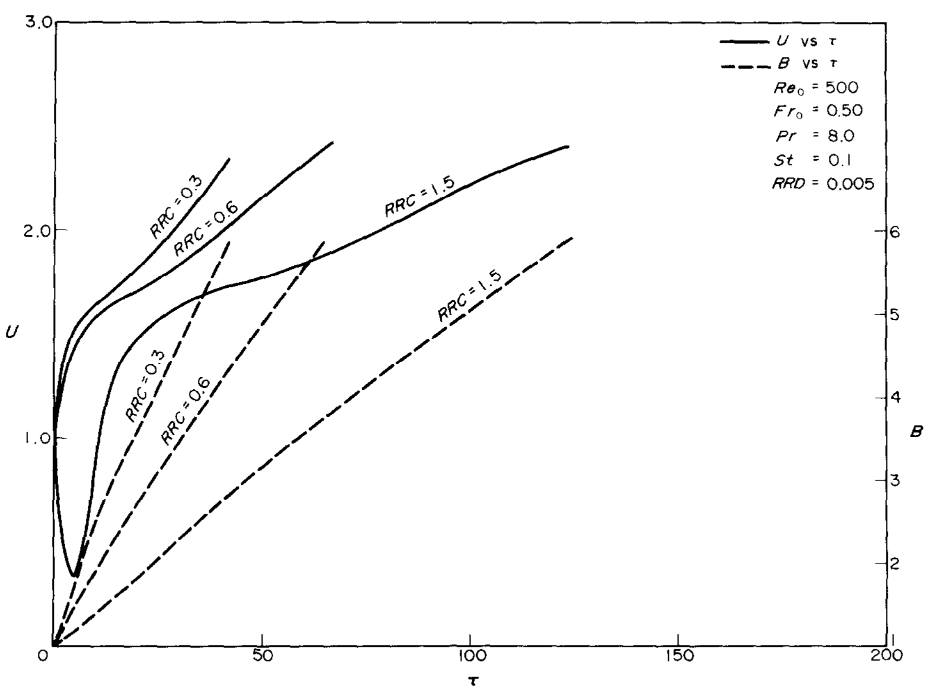


FIG. 7. Effect of RRC on U and B.

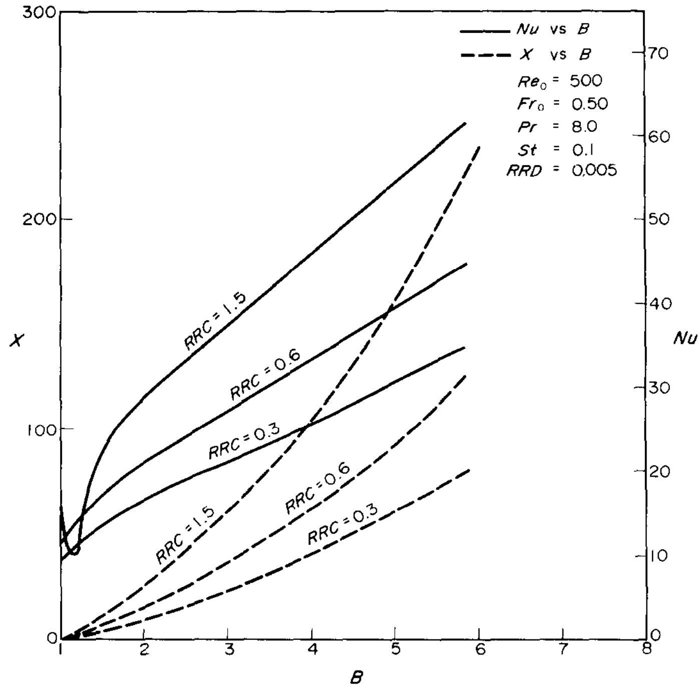


FIG. 8. Effect of *RRC* on *Nu* and *X*.

The ranges of the parameters are so selected as to cover different liquid pairs and to include all possible operating conditions of practical interest. From this it is possible to predict the process of evaporation of a droplet in any immiscible liquid.

Figures 3 and 4 show the effect of Prandtl number. Figure 3 shows the variation of B and U with τ. A higher Prandtl number decreases the bubble growth rate as is evident from equation (10) and hence increases the time for complete evaporation. A higher Prandtl number decreases the bubble acceleration, but the final velocity is slightly higher for higher Prandtl numbers. Figure 4

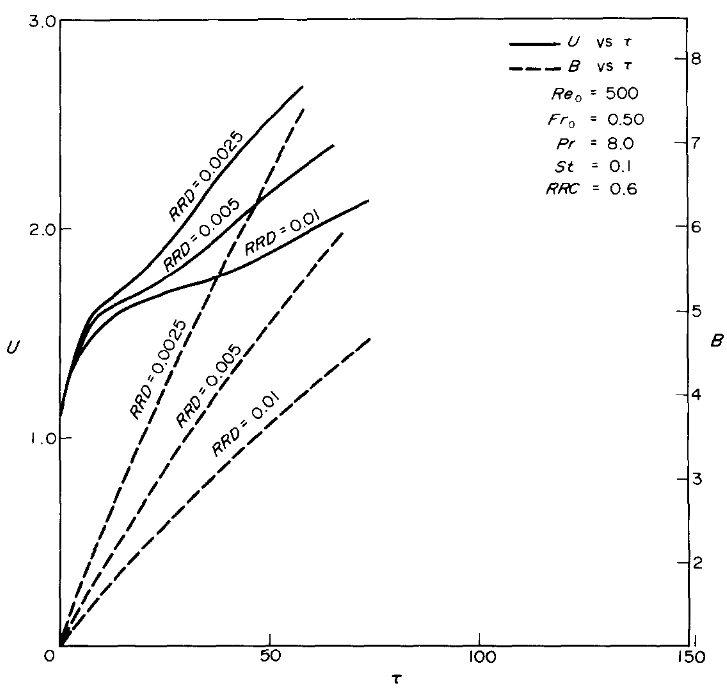


FIG. 9. Effect of *RRD* on *U* and *B*.

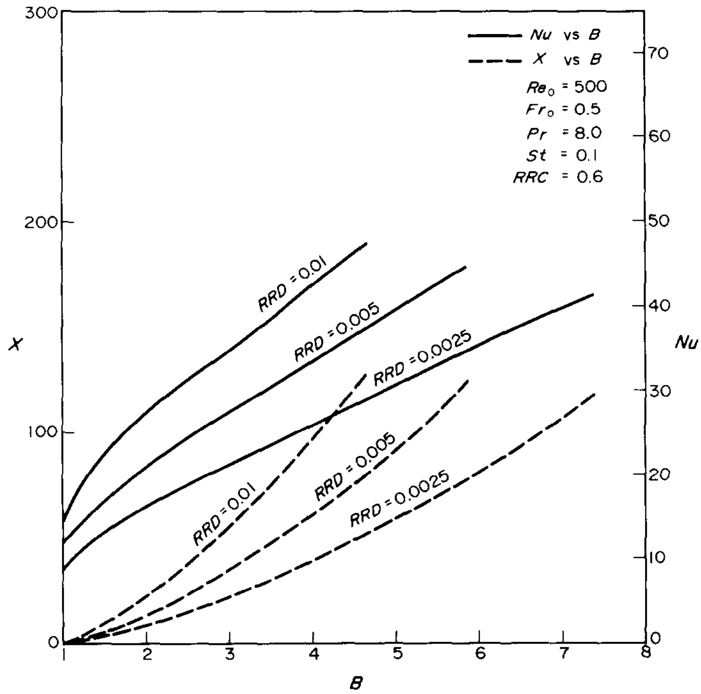


FIG. 10. Effect of RRD on Nu and X.

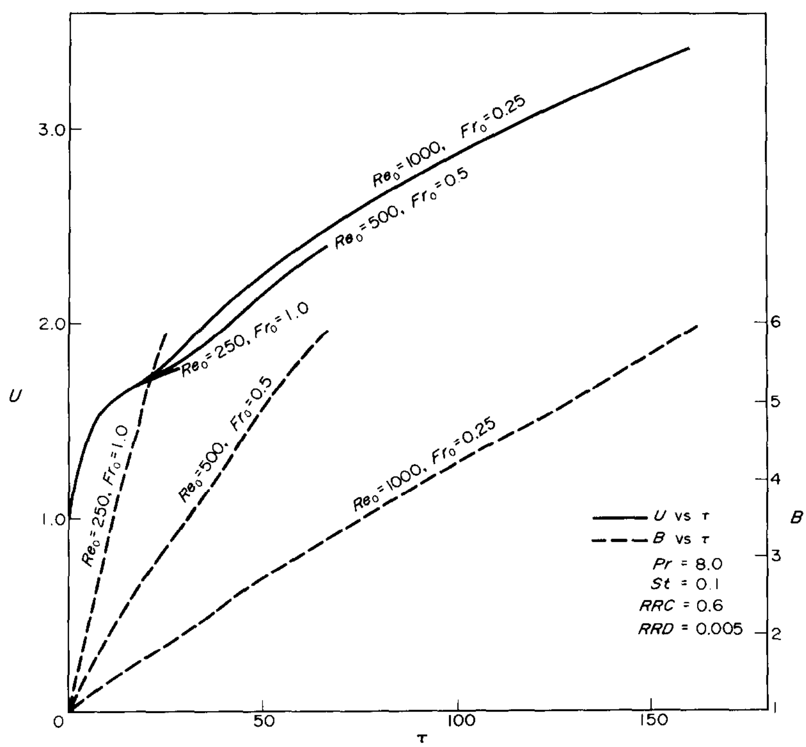


FIG. 11. Effect of initial drop diameter on U and B.

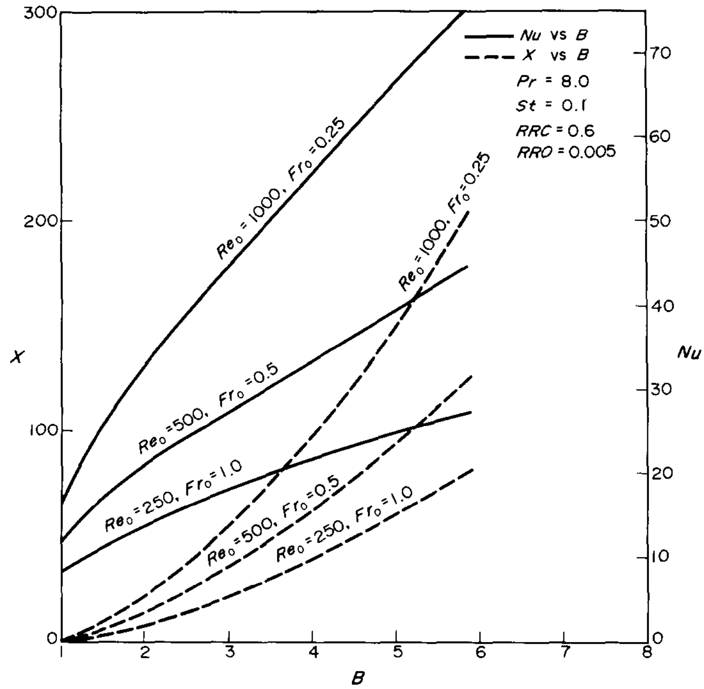


FIG. 12. Effect of initial drop diameter on Nu and X.

shows the variation of Nu and X with B. A higher Prandtl number results in a higher value of Nu and this is evident from equations (13) and (17) also. But the height required for complete evaporation is more at higher Prandtl numbers because of the lower bubble growth rates.

Figures 5 and 6 indicate the importance of system Stefan number. A higher Stefan number results in a corresponding increase in Jakob number. A higher Jakob number increases the bubble growth rate and reduces the Nusselt number. A higher bubble growth rate is accompanied by a higher acceleration of the drop, lower time and height for complete evaporation. But the final velocity increases slightly at lower Stefan numbers.

Figures 7 and 8 show the effect of the density ratio RRC. A lower value of RRC increases the Jakob number and the effect is almost similar to that of an increase in St. But the value of RRC affects the evaporation process at the initial stages. Figure 7 shows that at $RRC = 1.5$, meaning a dispersed phase liquid denser than the continuous phase liquid, the drop undergoes a deceleration and attains a velocity lower than the initial velocity and then it begins to accelerate. Figure 8 shows a corresponding decrease in Nu in the initial stage at $RRC = 1.5$. However, this effect is not felt above a radius ratio of about 1.5.

The effect of the density ratio RRD is shown in Figs. 9 and 10. A lower value of RRD increases the Jakob number and the effect is again almost similar to that of an increase in St. But the density ratio RRD determines the radius ratio at the completion of evaporation. The radius ratios at the completion of evaporation are 4.65, 5.85 and 7.37 for $RRD = 0.01$, 0.005 and 0.0025, respectively. The density ratio RRD also affects the final velocity. But its effect on the height for complete evaporation is not very considerable.

The effect of initial diameter is shown in Figs. 11 and 12. The Nusselt number is higher for larger drops. But the bubble growth rate is higher for smaller drops. The acceleration of the drops is almost the same for different diameters. But the final velocity attained depends on the initial drop diameter. The final velocity is less for smaller drops. Doubling the initial diameter increases the time for complete evaporation by about 2.5 times and the height for complete evaporation by about 3 times.

The effect of initial velocity on the total evaporation process is not significant and therefore is not shown here.

CONCLUSIONS

Parametric analysis of single drop evaporation in a stagnant column of immiscible liquid is carried out. The predicted results compare favourably with the available experimental results. From the analysis the following conclusions are made:

(1) The bubble growth rate, Nusselt number and the time and height required for complete evaporation depend mainly on the Péclet and Jakob numbers.

(2) The acceleration of a given drop during evaporation depends on the bubble growth rate.

(3) For a given initial diameter of the drop the final velocity is only slightly affected by changes in temperature difference.

(4) Larger drops attain higher final velocities.

(5) The ratio of the densities of the dispersed phase vapour and liquid determines the radius ratio at the completion of evaporation.

(6) The initial acceleration or deceleration of the drop depends on the ratio of the densities of the continuous and dispersed liquid phases.

(7) The parametric studies presented here help in the prediction of the process of evaporation of a drop in any immiscible liquid.

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ETUDES PARAMETRIQUES SUR L'EVAPORATION PAR CONTACT DIRECT D'UNE GOUTTE DANS UN LIQUIDE NON MISCIBLE

Résumé—On analyse théoriquement l'évaporation par contact direct d'une goutte en déplacement dans une colonne stagnante de liquide non miscible. Les paramètres adimensionnels gouvernant le mouvement et le transfert thermique sont identifiés et on étudie les effets de la variation de ces paramètres. L'accord entre les résultats du calcul et les données expérimentales disponibles est raisonnablement satisfaisant.

EINE PARAMETERSTUDIE DER DIREKTEN KONTAKTVERDAMPFUNG EINES TROPFENS IN EINER NICHTMISCHBAREN FLÜSSIGKEIT

Zusammenfassung—Es wird über eine theoretische Betrachtung der direkten Kontaktverdampfung eines Tropfens, der sich in einer ruhenden, nichtmischbaren Flüssigkeitssäule bewegt, berichtet. Die maßgebenden dimensionslosen Parameter der Bewegung und des Wärmetransports wurden identifiziert und Parameterstudien durchgeführt. Die Übereinstimmung zwischen den berechneten Ergebnissen und den verfügbaren experimentellen Werten ist gut.

ИССЛЕДОВАНИЕ ПРОЦЕССА КОНТАКТНОГО ИСПАРЕНИЯ КАПЛИ ПРИ ДВИЖЕНИИ В НЕСМЕШИВАЮЩЕЙСЯ ЖИДКОСТИ

Аннотация—Работа посвящена теоретическому анализу процесса контактного испарения капли, движущейся в неподвижном столбе несмешивающейся жидкости. Установлены безразмерные параметры, характеризующие этот процесс, и исследовано их влияние на тепломассоперенос. Согласие между результатами расчета и полученными экспериментальными данными достаточно хорошее.